Differing Interactions Require Baryon and Lepton Conservation

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Baryon and lepton numbers are conserved. Why? Baryon number must be because baryons are subject to strong interactions, leptons are not. Conservation of baryons leads to that of leptons. This raises further questions which are noted.

A fundamental attribute of elementary particles is their set of conservation laws. Some arise from the nature of space [ref. 1, p.27, 299], but others are nongeometrical, such as conservation of baryons, and following from it, conservation of leptons. Conservation of baryon, so of lepton, numbers, comes not from space but from their interactions.

This discussion is schematic, excluding everything unneeded, which simplifies the writing and aids understanding, and by not including the irrelevant, emphasizes the generality of results. We ignore electromag- netism and gravitation and their labels. They do not allow decay of protons. To be concrete, and as it is often discussed this way, we speak of the decay of the proton, but this applies to any baryon (a fermion with strong interactions), and antibaryon, and pions, but that can be any meson (a boson with strong interactions). Also lepton (a fermion with no strong interactions) means either lepton or antilepton. All three types of particles have weak interactions, that would cause, if it were possible, the proton to decay into leptons plus pions, or directly into leptons (it does not matter if the decay is attributed to other interactions; the label weak merely denotes the interaction, whatever it is, whatever its properties, that causes the decay that we wish to show is impossible). Strong interactions are often stated as plural, for generality. As there are no other stable particles (known), the analysis indicates that if there are several types, every baryon is affected by each.

Why is baryon number conserved? If it were not a particle with strong interactions would go into ones without; strong interactions would be "turned off". But weak interactions cannot "turn off" strong ones. There is no Hamiltonian with one interaction that changes another interaction. We first give an intuitive picture, then a formal argument. Purely heuristically, to see what goes wrong, take the unphysical case of the proton having weak interactions, the pion not. This gives a Feynman diagram in which the proton emits a virtual pion and then decays into three lep- tons. But the pion, being virtual, has to be reabsorbed. The lep- tons, however, do not interact with it, so it cannot be reabsorbed leaving it in a very unpleasant situation — implying that protons cannot so decay. Similarly, a state only of leptons, with no strong interactions, cannot go into one with baryons (without an equal number of antibaryons). The reac- tion is not possible in either direction.

While the weak interaction of baryons cannot turn the strong one off or on, mesons do go into leptons. Why? The photon is analogous; its number is not conserved, though charge is. However it is neutral, it couples to a neutral object, taken as a particle-antiparticle pair (the current is of this form). Electron-photon scattering can be regarded as the creation of a pair by the photon; the positron annihilates the electron, and the electron of the pair replaces it. So while the electromagnetic interaction (of an elec- tron) cannot be altered, photons can be taken as not having a direct inter- action. Their creation or annihilation does not modify an interaction. Similarly for mesons, which can be deemed to have no strong interactions, but rather couple to particle-antiparticle

pairs. We can view meson-baryon scattering as annihilation of the baryon by the antiparticle of the pair, and its replacement by the baryon of the pair. Because of what they are coupled to, mesons can decay into objects not affected by strong interac- tions.

With this intuitive understanding of why baryons cannot decay into only leptons, but mesons can, we turn to a formal analysis. To study the decay we consider the action of the Hamiltonian on a proton. Acting on a state at t = 0, time-translation operator exp(iHt), H is the Hamiltonian, gives the state at time t. Can this take a baryon to a state whose fermions are only those not having strong interactions? The effect of this is seen from that of H, a sum of the free particle Hamiltonians for the proton, pion and leptons, plus interaction terms, for the weak and strong interactions. The state of the system is a sum of terms, one the state of the proton, another (if decay were possible) the product of pion and lepton states (plus products of lepton states if there were those decays), each sums over other labels and integrals over momenta or space. Take the initial state as a proton, say at rest. The free part of H changes its phase. The weak interaction part, were decay possible, decreases its coef- ficient in the sum, while increasing that of the (say) pion plus lepton, initially zero — starting as a proton, the state becomes a sum of the proton, its contribution decreasing, plus the pion plus lepton state, with increasing contribution. For the decaying pion the behavior is similar: starting as a pure pion it becomes a sum of that plus a state of leptons, with the contribution of the first decreasing, of the second increasing.

What is a proton? We define it as a particle obeying Dirac's equation, with mass m(P), where this equation includes the (irrelevant and suppressed) electromagnetic, weak and strong interactions (whose forms are not needed). It is the presence of these interactions that determines what a proton is. (Correctly a particle is an eigenstate of the two Poincaré invariants [ref. 2, p.114]. For a free particle, and one with an electromagnetic interaction, Dirac's equation is equivalent. Whether this is true with weak and strong interactions seems unknown so consequences of, perhaps important, differences, if any, are not clear. And it is possible that put- ting interactions in invariants, which must be done whether Dirac's equa- tion is used or not, might limit them. Particles are also eigenstates, or sums, of the momentum operators [ref. 3, p.93], of which the Hamiltonian is one. We ignore these, and refer to Dirac's equation as it is familiar, but the discussion could be of invariants, which might be revealing). The statefunction of the proton then is a solution to coupled nonlinear equations. We need information about it but cannot solve explicitly so represent it in a way that allows analysis, using an expansion. The argu- ments though are exact; we do not calculate, so need not truncate.

The physical particle, labeled with a capital, that obeying Dirac's equa- tion with all interactions, is a sum of states (schematically): summing over all states to which the proton is connected by interactions, including any number of pions, and so on. The summations represent ones over internal labels and integrals over

momenta. These, and all other irrelevancies, like spin, are suppressed. That they can be shows the generality of this. State p) is the function satisfying Dirac's equation with the weak interaction, but not the strong — its effect is given by this sum, which is thus an eigenstate, with mass m(P), of the total Hamiltonian, including all interactions. Individual terms in the sum differ in energy and momenta; it is the sum that has the eigenvalues. We can take pi) as either bare or physical; for the former each term is an infinite sum, which is encapsulated by regarding it as physical.

The coefficients are determined by the requirement that this be a solution of the complete Dirac's equation, and normalization (P P) = 1. Also the initial state, say a proton at rest, gives c(x,0), a wavepacket, with all other c's = 0, at t = 0; they depend on the statefunction for P). These particles are virtual in not obeying the physical Dirac's equation, that with interactions. Only P is physical. Dirac's equation (with interactions schematic, and higher-order terms not excluded) is

States of different numbers of pions are orthogonal, so this gives an infinite set of coupled equations for the c's; solving (in principle) gives the state of the (physical) proton. (We need not consider how far this can be taken to find, and solve, recursion relations to obtain physical states; doing so for a particle that does decay might provide information about it, and its other interactions.) This expansion has an infinite number of pions, that is terms p)pi)**r, for all r geq 0, because (writing p) and pi) using creation and annihilation operators acting on the vacuum) the strong part of the interaction Hamiltonian is (schematically) is the annihilation, a* the creation, operators for p), the b's are the ones for pions. This acting on p)pi)**r gives p)pi)**(r+1) and p)pi)**(r-1), so the expan - sion includes all r geq 0.

Now the weak interaction acts on p) supposedly causing it to decay, so the final state is

showing the transition to, say, a pion plus a lepton, and to three leptons, with coefficients of non-occurring terms zero. The energy of fs) is m(P), not m(p), so needs contributions from sum c(p)pi) l)pi)pi), and so on. However fs) is, say, a lepton plus a pion, so these other terms, to which this is orthogonal, cannot contribute. The decay of the proton cannot conserve energy, thus cannot occur. Likewise decays of leptons (the tau) to baryons are similarly ruled out.

Contrast this with the strong decay of the Delta; the physical one is Delta(p), while Delta satisfies Dirac's equation without the strong interaction (thus with no interaction); pi(p) is the physical pion, pi satisfies interaction-free equations. Then Delta(p) has an expan- sion similar to the proton's, except that it can decay by the strong inter- action into a proton plus a pion giving extra terms (with coefficients w), The Delta), having no interac- tions, does not decay, so this expansion cannot be used for an argument like that for the proton's weak decay — p) and Delta) are solutions of the free-particle Dirac's equation, there is only a single interaction the strong one (which causes the decay), so the argument fails. This decay, of the entire sum, is possible. (But Delta) cannot

decay into particles with no strong interaction.)

The neutron has an actual weak decay; why do the arguments not apply? Taking it at t = 0 in a single particle state, gives obeying Dirac's equation with the weak interaction, but not the strong, and N) is the physical particle, with both interactions. The n) decays to p) l) l)', so this becomes The d- coefficients give the physical state P) l) l)'. The state then is the first coefficient decreases in time, the second increases, so the total probability is constant. These two (orthogonal) states have equal energy. The neutron can decay into a strongly-interacting particle (the proton), but not otherwise, because it is state n) that decays to p). But the physical neutron i s a sum of terms like n)pi)**r, so the resultant proton is a sum of such terms as p)pi)**r, which is the physical proton state — the proton, affected by the strong interaction, is such a sum (plus others appearing also for the neutron). If the neutron were to decay into state L) without the strong interaction, the expansion would have terms like L)pi)**r, which do not sum to a state of any physical particle — there is no final state with this expansion so the matrix element of the Hamiltonian causing such decay is zero; decays annulling the strong interaction can- not occur.

Conservation of baryons implies that of leptons. The numbers of fermions in all states must all be odd, or all even, and we assume that the number of particles minus antiparticles is constant (it is not clear that otherwise a hermitian Hamiltonian is possible that would not lead to difficulties like those above, especially if neutrinos have nonzero mass, as we expect [ref. 3, p.70]). Then leptons cannot be produced or destroyed if baryon (minus antibaryon) number is constant — lepton number is con- served.

The pion can decay into leptons: Why does the argument not apply? The physical pion pi(p)) can be written obeys Dirac's equation with strong interactions, but not the weak — this causes the decay — and the prime indicates antiparticles. This contains two terms (plus irrelevant ones for other particles coupled to the pion). The strong part of the pion's Hamiltonian acting on a pion gives a particle- antiparticle pair, acting on this pair gives the pion, and similarly for r pions which is mixed with the pair plus r-1 pions. Only two states appear in the expansion for the pion, unlike the infinite number for the proton. The weak part of H acting on pp') causes it to decay to two leptons; this is related by crossing to proton-antiproton scattering into two leptons, and also to the decay n Rightarrow p + 2l, which we saw is possible. Thus both terms in the sum for the physical particle con- tribute to the decay, it does conserve energy (and momentum), therefore cannot be ruled out.

The argument for electric-charge conservation is the same. Charge conservation is related to gauge invariance, a partial statement of Poincaré invariance [ref. 3, p.43] — this relates an allowed interaction to the Poincaré group. An interaction violating charge conservation would not transform under gauge transformations as other terms in the Hamiltonian, giving Poincaré transformations (on massive objects) that induce gauge transformations (on massless ones) resulting in physically-identical obser- vers who undergo the different gauge

transforma- tions — these cannot be fully specified — thus physically identical, but who see different Hamiltonians. The Hamiltonian would not be well-defined, implying inconsistent physics. It is fortunate that charge is conserved.

There are other implications requiring investigation; we mention them in hope of stimulating such.

All interactions known are of lowest order. Why? For electromagnetism linearity is enforced by gauge (Poincaré) invariance [ref. 3, p.57]. For strong interactions, take a particle, a Delta or p, that emits a pion. Higher-order terms would couple it not to a single pion, but to more. Intuitively we can guess why only lowest order occurs since it gives diagrams which we interpret (purely heuristically) as two, or more, pions emitted sequentially. Higher-order means that these are emitted together. However this is the limit of the lowest order in which the time between emissions goes to zero. A higher-order interaction would be this limit, which is included in the lowest order as one case; higher-order terms adding nothing, would be irrelevant. Summing all diagrams, and integrating over time, would give contributions from terms that have the same effect as higher-order ones, thus changing only the value of the sum, so the value of the coupling constant — an experimental parameter (at present), thus we could not distinguish contributions from terms of different order, implying higher order would be undetectable. This regards particles as virtual. But consider a decay in two steps, each emitting a pion. If the intermediate object's life were sufficiently short, this would be equivalent to pions being emitted simultaneously. If a nucleon had an interaction of the form NNN' pi, the emission of an NN' pair could be thought of as due to the decay of a pion, and the interaction taken as the limit of the emission of a pion, and then its decay, when its lifetime becomes zero, merely changing the sum.

Suppose that the only particles were nucleons and pions, no excited ones. With a linear interaction final states of pion-nucleon scattering have only a single pion — the nucleon could not store the extra energy and decay to a second pion. Nonlinear interactions give states with more than one pion, but could be simulated by short-lived excited states — the pion scatters and excites the nucleon which then decays giving a second pion. Nonlinear interactions would be indistinguishable from existence of excited states; perhaps we could require all interactions be linear with addition of excited states. Why is the gravitational interaction of lowest order? The interaction of matter with gravity is clear [ref. 3, p.73]. But what determines how matter fixes the gravitational field [ref. 3, p.150]? The tensor could be T(mu nu)T(nu rho), or could it? These are some questions that should be looked at.

Gravitation raises another point. There are reasons why it may be unable to act on scalar particles: pions, kaons, ... [ref. 3, p.61]. Assuming that it acts on vectors, like rho, then objects affected by gravity decay to ones that are not (the rho to pions), and ones not affected go to ones that are (the pion to leptons). Does the argument not forbid this? It fails because there is no such thing as a state of, say, two gravitons [ref. 3, p.187], nor even a free graviton,

since gravity is — necessarily — non- linear [ref. 3, p.69]. The question remains open. Except for a few particles, it is not even known (experimentally) which have interactions with gravity, nor what these are — something of great interest. Theoreti- cal prejudice should not substitute for analysis and experiment. This emphasizes the difference between gravity and other interactions, and the value of these questions as a probe into the laws of nature.

There are strong constraints on physics as seen in many ways [ref. 1, p.2; ref. 2; ref. 3]. It is well known that interactions are greatly limited by the properties of space, but also because of other interactions. Perhaps this analysis, and more the questions it leads to, can induce further inquiry. There are reasons for the laws of nature, and it is fortunate that reasons, and laws, are such that we are able to find, and understand, them.

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